Admittance Matrix Concentration Inequalities for Understanding Uncertain Power Networks

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Power grids themselves are often not known for certain

Storms take lines out without warning.


Photo: A. Kwasinski (UPitt)

Hurricane Ike

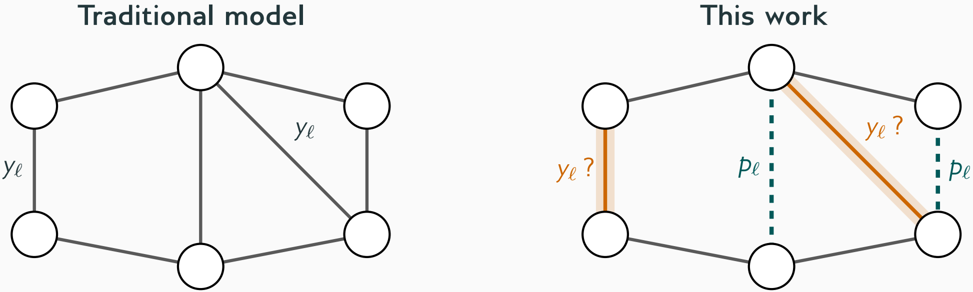
New forms of risk.


Photo: UAV Coach

Drones+transmission lines=?

Also measurement error in line data, power flow control devices, and deliberate attacks.

The network itself can be uncertain



Known lines, known admittances: Y fixed.

Uncertain lines and admittances: Y random.

The admittance matrix becomes random.

Uncertainty is familiar to our field

Perspective	Random object	Typical question
Probabilistic power flow	Power injections	How do injections affect the state?
Cascading failures	Sequence of line failures on a graph	How do failures spread?
This work	**Complex admittance matrices**	**How does network uncertainty affect power flow?**

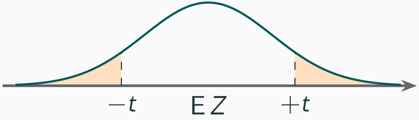
Random line data → Admittance spectrum → Power flow guarantees

Probabilistic power flow: Borkowska, 1974. Cascading failures: Watts, PNAS 2002; Hines, Cotilla-Sanchez, and Blumsack, Chaos 2010. Random Laplacians: Oliveira, arXiv:0911.0600; Le, Levina, and Vershynin, 2017.

A concentration inequality controls rare deviations

Scalar concentration

$$\Pr\big(|Z - \mathrm{E}\, Z| \geq t\big) \leq \delta(t).$$



Chance of straying farther than t

Matrix concentration

$$\Pr\big(\|Y - \mathrm{E}\, Y\|_2 \geq t\big) \leq \delta(t).$$

The operator norm is the largest response to any unit input:

$$\|M\|_2 = \sup_{\|x\|_2=1} \|Mx\|_2 \,.$$

A tail bound also bounds the **average**, since for $W \geq 0$, $\mathrm{E}\, W = \int_0^\infty \Pr(W > t)\, dt$.

Matrix Bernstein bounds a sum of independent matrices

Independent, symmetric $X_1, \dots, X_k \in \mathbb{R}^{d \times d}$ with $\mathrm{E}\, X_\ell = 0$ and $\|X_\ell\|_2 \le R$.

$$X = \sum_{\ell=1}^{k} X_\ell, \qquad \nu = \left\| \sum_{\ell=1}^{k} \mathrm{E}[X_\ell^2] \right\|_2.$$

Matrix Bernstein

$$\Pr\left(\|X\|_2 \ge t\right) \le 2d \exp\left(-\frac{t^2/2}{\nu + Rt/3} \right), \qquad t > 0.$$

$$\mathrm{E}\, \|X\|_2 \le \sqrt{2\nu \log(2d)} + \frac{R}{3} \log(2d).$$

R: largest single term $\quad$ ν: total variance $\quad$ d: matrix size

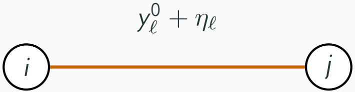
J. A. Tropp, *An Introduction to Matrix Concentration Inequalities*, Found. Trends Mach. Learn., 2015.

Two models of random line admittances

Fixed lines, random admittances

$$w_\ell = y_\ell^0 + \eta_\ell, \qquad \mathrm{E}\,\eta_\ell = 0, \quad |\eta_\ell| \le r.$$

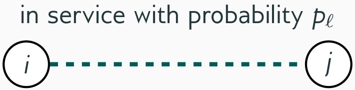
- Independent line perturbations η_ℓ.
- Any bounded distribution.
- Bounded and zero mean, hence sub-Gaussian.



Fixed admittances, random lines

$$w_\ell = y_\ell \xi_\ell, \qquad \xi_\ell \sim \mathrm{Bernoulli}(p_\ell).$$

- Independent on/off statuses ξ_ℓ.
- Each line has its own probability p_ℓ, so **inhomogeneous Erdos–Rényi** graph.



where $Y = \sum_\ell w_\ell E_\ell, \quad \widetilde{Y} := Y - \mathrm{E}\,Y.$

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Theorems 2–3**.

Decompose the admittance into independent line matrices

Line $\ell = (i, j)$ contributes the elementary Laplacian

$$E_\ell = (e_i - e_j)(e_i - e_j)^\top, \qquad E_\ell\big|_{\{i,j\}} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

With $w_\ell = g_\ell + \mathrm{j}b_\ell$ and $Y = G + \mathrm{j}B$, write Y as one real symmetric matrix, the lift

$$\boxed{\mathcal{H}(Y) := \begin{bmatrix} G & B \\ B & -G \end{bmatrix} = \sum_{\ell \in E} \begin{bmatrix} g_\ell & b_\ell \\ b_\ell & -g_\ell \end{bmatrix} \otimes E_\ell.}$$

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Section II**. Undirected model with no shunts or taps.

A bound for any bounded line distribution

n buses, at most Δ lines at any bus. Independent zero mean line errors η_ℓ with $|\eta_\ell| \leq 1$.

Theorem 2: bounded admittance errors on fixed lines

$$\mathrm{E}\left\|\widetilde{Y}\right\|_2 \leq \sqrt{8\Delta\log(4n)} + \frac{2}{3}\log(4n).$$

- Any independent, bounded line errors; no distribution shape assumed.
- Only logarithmic growth in the number of buses n.
- Errors up to r: multiply the bound by r.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Theorem 2**. Applied to the centered admittance perturbation.

For contingencies, variance accumulates at buses

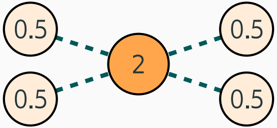
Fixed $|y_\ell| \le 1$; independent $\xi_\ell \sim \mathrm{Bernoulli}(p_\ell)$.

$$\underbrace{c_\ell = 2p_\ell(1-p_\ell)|y_\ell|^2}_{\text{line}} \quad \underbrace{d_i(c) = \sum_{\ell \ni i} c_\ell}_{\text{bus}} \quad \underbrace{\Delta_c = \max_i d_i(c)}_{\text{largest bus}} \quad \underbrace{\bar{D} = \frac{\sum_i d_i(c)}{\Delta_c} \le n}_{\text{normalized total}}$$

Spread along a path

$\Delta_c = 1, \qquad \bar{D} = 4$

Concentrated at a hub



$\Delta_c = 2, \qquad \bar{D} = 2$

Same four lines with $c_\ell = 1/2$: the hub doubles Δ_c and halves $\bar{D}$.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Definition 3**. Contingency factors and nodal criticality.

$\widetilde{Y} = \sum_{\ell \in E} (\xi_\ell - p_\ell) y_\ell E_\ell$; independent on/off statuses, $|y_\ell| \leq 1$, $\Delta_c > 0$.

Theorem 3: independent random line outages

For $t \geq \sqrt{2\Delta_c} + 2/3$,

$$\Pr\left(\left\|\widetilde{Y}\right\|_2 \geq t\right) \leq 8\bar{D} \exp\left(-\frac{t^2}{4(\Delta_c + t/3)}\right).$$

$$\mathrm{E}\left\|\widetilde{Y}\right\|_2 \leq C\left(\sqrt{2\Delta_c \log(1 + 2\bar{D})} + 2\log(1 + 2\bar{D})\right).$$

with a universal constant $C > 0$.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Theorem 3**. Intrinsic dimension Bernstein.

The flat start Jacobian shares the admittance matrix norm

Lifted admittance $\mathcal{H}(Y)$

$$\mathcal{H}(Y) = \begin{bmatrix} G & B \\ B & -G \end{bmatrix}.$$

Flat start Jacobian F

$$F = \begin{bmatrix} G & -B \\ -B & -G \end{bmatrix}.$$

$Q = \operatorname{diag}(I_n, -I_n)$ flips the sign of the block and preserves its norm $F = Q\mathcal{H}(Y)Q$.

$$\boxed{\|F\|_2 = \|Y\|_2\,, \qquad \|F - \mathrm{E}\,F\|_2 = \|Y - \mathrm{E}\,Y\|_2\,.}$$

$$\underbrace{\begin{bmatrix} p \\ q \end{bmatrix} \approx F \begin{bmatrix} v-1 \\ \theta \end{bmatrix}}_{\text{Linear Coupled Power Flow (LCPF)}} \qquad \underbrace{v - 1 \approx Rp + Xq}_{\text{LinDistFlow sensitivity form}} \;.$$

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Section II and Appendix A**.

A family of networks gives a family of AC manifolds

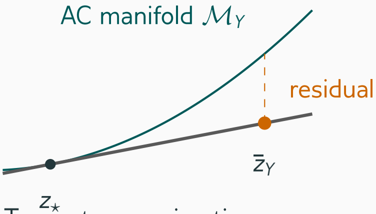
Complex voltages $u \in \mathbb{C}^n$ in rectangular coordinates, injections $s \in \mathbb{C}^n$

$$\Phi_Y(u) = \operatorname{diag}(u)\overline{Yu}, \qquad \mathcal{M}_Y = \{(u, \Phi_Y(u)) : u \in \mathbb{C}^n\} \subset \mathbb{R}^{4n}.$$

From $z_\star = (u_\star, \Phi_Y(u_\star))$, a step h along the **tangent** is

$$\bar{z}_Y = z_\star + (h, D\Phi_Y(u_\star)[h]).$$

Bound the average distance to the AC manifold over random networks.



Tangent approximation

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Section IV**.

Average tangent error is quadratic in the voltage step

Fixed lines, with the independent zero mean line errors of Theorem 2 ($|\eta_\ell| \leq 1$).

Proposition 1: expected distance to the AC manifold

$$
\begin{aligned}
\mathrm{E}\,\mathrm{dist}(\bar{z}_Y, \mathcal{M}_Y) &\leq 3\|h\|_\infty \|h\|_2 \;\mathrm{E}\,\|Y\|_2 \\
&\leq 3\,\|h\|_2^2 \left(\sqrt{8\Delta \log(4n)} + \frac{2}{3}\log(4n)\right).
\end{aligned}
$$

For a physical network $Y = Y_0 + \widetilde{Y}$, keep the nominal term $\mathrm{E}\,\|Y\|_2 \leq \|Y_0\|_2 + \mathrm{E}\left\|\widetilde{Y}\right\|_2$.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Proposition 1**.

Random switching adds a probabilistic curvature term

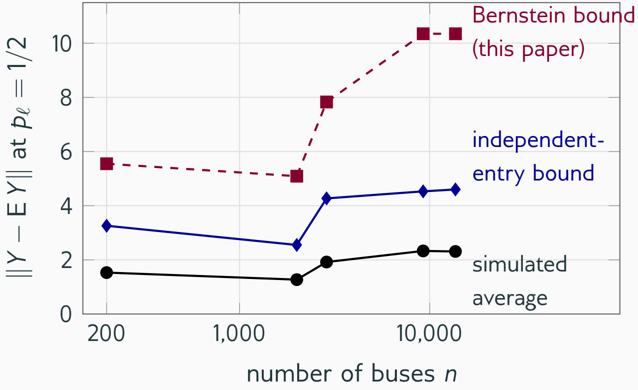
Lossless network $Y = -\mathrm{j}L_\beta$, $L_\beta = \sum_\ell \beta_\ell \xi_\ell E_\ell$, $0 < \beta_\ell \le 1$. Independent $\xi_\ell \sim \mathrm{Bern}(p_\ell)$.

Proposition 2: AC manifold distance under random line outages

$$\mathsf{E}\ \mathrm{dist}(\bar{z}_Y, \mathcal{M}_Y) \le 3\|h\|_\infty \|h\|_2 \left[C \left(\sqrt{2\Delta_c \log(1+2\bar{D})} + 2\log(1+2\bar{D}) \right) \right].$$

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Proposition 2**.

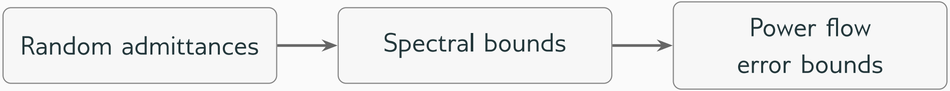
Measured deviations stay modest from 200 to 13,659 buses



- Five transmission test cases, 200 to 13,659 buses.
- Each line in service independently with $p_\ell = 1/2$; $\max_\ell |y_\ell| = 1$.
- Simulated average between 1.3 and 2.3; the bounds sit $2\times$ to $4.5\times$ above it.

Independent entry bound from A. S. Bandeira and R. van Handel, *Sharp nonasymptotic bounds on the norm of random matrices with independent entries*, Ann. Probab., 2016.

Concentration supports a broader probabilistic analysis



- **Analyze existing algorithms:** average error across a family of networks.
- **Design randomized algorithms:** sampling tailored to a problem, with guarantees.
- **Study grid decisions:** probabilistic operations, reliability, and planning.

These directions are becoming new results.

Two developments since this work began

Randomized switching

Design switching probabilities with controlled reconfiguration cost.

Efficient Network Reconfiguration by Randomized Switching

Quantum PF limits

Grid structure makes the equations ill-conditioned, which limits quantum speedups.

Proving the Limits of Quantum Power Flow

Thank you! Questions

Backup slides

The line decomposition gives the Bernstein parameters

For a centered line error η_ℓ, $X_\ell = \begin{bmatrix} \operatorname{Re}\{\eta_\ell\} & \operatorname{Im}\{\eta_\ell\} \\ \operatorname{Im}\{\eta_\ell\} & -\operatorname{Re}\{\eta_\ell\} \end{bmatrix} \otimes E_\ell$.

$$E_\ell^2 = 2E_\ell, \qquad X_\ell^2 = 2|\eta_\ell|^2 I_2 \otimes E_\ell, \qquad \|X_\ell\|_2 = 2|\eta_\ell|.$$

Bounded weights: $|\eta_\ell| \leq 1$.

$$\nu \leq 2 \left\| \sum_\ell E_\ell \right\|_2 \leq 4\Delta.$$

Use $d = 2n$, $R = 2$ in Theorem 1.

Switches: $\eta_\ell = (\xi_\ell - p_\ell) y_\ell$.

$$V = \sum_\ell c_\ell E_\ell, \qquad \nu = \|V\|_2 \leq 2\Delta_c.$$

The intrinsic dimension obeys $2\dfrac{\operatorname{tr} V}{\|V\|_2} \leq 2\bar{D}$.

The variance is a positive weighted Laplacian even when the summands are indefinite.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Appendices B–C**. Back to Slides 8 and 10.

Centering separates nominal curvature from random variation

In rectangular coordinates the exact tangent residual is

$$r_Y(h) = \Phi_Y(u_\star + h) - \Phi_Y(u_\star) - D\Phi_Y(u_\star)[h] = \operatorname{diag}(h)\overline{Yh}.$$

Write $Y_0 = \mathrm{E}\, Y$. At a common fixed step h,

$$\begin{aligned} \mathrm{E}\,\|r_Y(h) - r_{Y_0}(h)\|_2 &\le \|h\|_\infty \, \|h\|_2 \; \mathrm{E}\,\|Y - Y_0\|_2\,, \\ \mathrm{E}\,\|r_Y(h)\|_2 &\le \|h\|_\infty \, \|h\|_2 \left(\|Y_0\|_2 + \mathrm{E}\,\|Y - Y_0\|_2\right). \end{aligned}$$

The same voltage AC point gives $\operatorname{dist}(\bar{z}_Y, \mathcal{M}_Y) \le \|r_Y(h)\|_2$.

If every line is deterministic, the fluctuation term vanishes; nominal AC curvature remains.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Propositions 1–2**. Clarification. Back to Slides 13–14.

On a tree, the inverse yields LinDistFlow sensitivities

Delete the slack bus column from the branch-to-bus incidence matrix of a tree: A_r is square and invertible. With line impedances $z_\ell = r_\ell + \mathrm{j}x_\ell$,

$$Y_r = A_r^\top \operatorname{diag}(1/z_\ell) A_r, \quad Y_r^{-1} = A_r^{-1} \operatorname{diag}(z_\ell) A_r^{-\top} = R + \mathrm{j}X.$$

$$\begin{bmatrix} v - \mathbf{1} \\ \theta \end{bmatrix} \approx \begin{bmatrix} R & X \\ X & -R \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}.$$

The first block row is the voltage magnitude LinDistFlow relation.

For squared magnitudes, $v^2 - \mathbf{1} \approx 2(Rp + Xq)$.

S. Talkington et al., *Admittance Matrix Concentration Inequalities*, PowerUp 2026, **Appendix A**. Unit slack voltage, no shunts or taps. Back to Slide 11.